

① Don't know σ^2 so must use t

From calculator:

$$n = 8$$

$$\bar{x} = 39.5$$

$$\sum x = 316$$

$$s = 4.8403\dots$$

$$\sum x^2 = 12,646$$

$$s^2 = 23.4285\dots$$

$$v = 8 - 1 = 7$$

$$t_{(7)} \text{ for } 95\% = 2.365 \quad (\text{look up } 0.975)$$

$$95\% \text{ CI} = 39.5 \pm 2.365 \times \frac{4.8403\dots}{\sqrt{8}}$$

$$= 39.5 \pm 4.0472\dots$$

$$= (35.5, 43.5)$$

② a) i) $A \sim \text{Po}(3.5)$

$$P(A=4) = \frac{e^{-3.5} \times (3.5)^4}{4!} = 0.18891\dots$$

ii) $B \sim \text{P}(5)$

$$P(B \leq 6) = 0.7622 \quad \text{from tables}$$

iii) $A+B \sim \text{Po}(8.5)$

$$P(A+B < 10) = P(A+B \leq 9) \\ = 0.6530$$

b) Let $(A+B) = X \rightarrow X \sim \text{B}(5, 0.6530)$

$$P(X \geq 4) = P(X=4) + P(X=5)$$

$$= {}^5C_4 \times (0.653)^4 \times (0.347) + {}^5C_5 \times (0.653)^5$$

$$= 0.31547 + 0.11873$$

$$= 0.434 \quad (30p)$$

c) i) From calc: $\bar{x} = 9.2$ $s = 3.04716$ $s^2 = 9.2898\dots$

ii) Mean and variance have similar values (9.2, 9.28...), which suggests a Poisson may be appropriate.

③

$$H_0: \mu = 85.9$$

$$H_1: \mu \neq 85.9 \quad (2 \text{ tailed})$$

$$\sum x = 8350$$

$$n = 100$$

$$\sum (x - \bar{x})^2 = 15321$$

$$\bar{x} = \frac{8350}{100} = 83.5$$

$$s^2 = \frac{15321}{99} = 154.7575\dots \quad s = 12.440\dots$$

Over 30 people, so use Z distribution because of CLT

5%, 2 tailed test

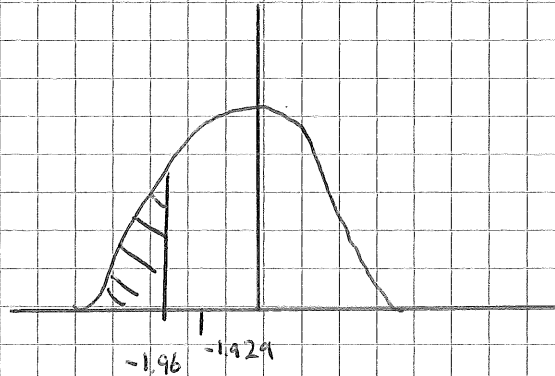
$$\text{TEST STATISTIC: } Z = \frac{83.5 - 85.9}{\frac{12.440}{\sqrt{100}}} = -1.929$$

CRITICAL VALUE: 5%, 2 tailed test = ± 1.96

$$-1.929 > -1.96$$

\therefore Accept H_0

Not enough evidence at 5% level to suggest average golf scores have changed.



④ a) Probs must sum to 1

$$0.62 + k = 1 \quad \rightarrow \quad k = 0.38$$

$$\begin{aligned} b) i) E(X) &= 1 \times 0.01 + 2 \times 0.05 + 3 \times 0.14 + 4 \times 0.3 \\ &\quad + 5 \times 0.38 + 6 \times 0.12 = 4.35 \end{aligned}$$

$$\begin{aligned} ii) E(X^2) &= 1^2 \times 0.01 + 2^2 \times 0.05 + 3^2 \times 0.14 + 4^2 \times 0.3 \\ &\quad + 5^2 \times 0.38 + 6^2 \times 0.12 = 20.09 \end{aligned}$$

$$\begin{aligned} \text{Var}(X) &= E(X^2) - E(X)^2 \\ &= 20.09 - (4.35)^2 = 1.1675 \end{aligned}$$

$$\begin{aligned} c) i) E(Y) &= 5E(X) + 2 \\ &= 5 \times 4.35 + 2 = 23.75 \end{aligned}$$

$$\begin{aligned} \text{ii) } \text{Var}(Y) &= 5^2 \text{Var}(X) \\ &= 25 \times 1.1675 = 29.1875 \\ \therefore \text{sd}(Y) &= \sqrt{29.1875} = 5.4025 \dots \end{aligned}$$

5) a) $H_0: \mu = 30$
 $H_1: \mu > 30$

Small sample, no population variance, so use t

From calculator:

$$\begin{aligned} n &= 10 & \bar{x} &= 33.5 \\ \sum x &= 335 & s &= 4.2491 \dots \\ \sum x^2 &= 11385 & s^2 &= 18.0555 \dots \\ v &= 10 - 1 = 9 \end{aligned}$$

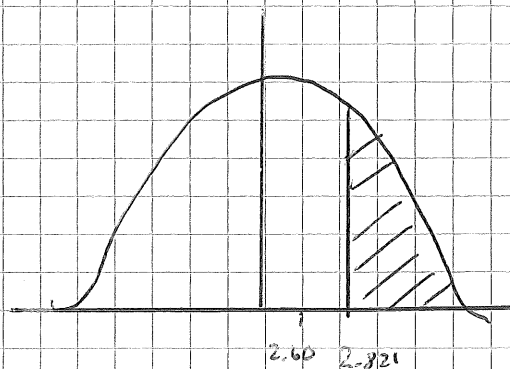
TEST STATISTIC: $t = \frac{33.5 - 30}{\frac{4.2491}{\sqrt{10}}} = 2.6047 \dots$

CRITICAL REGION: $t_{(9)}^{(1\%)} (1 \text{ tailed}) = 2.821$

$$2.60 < 2.821$$

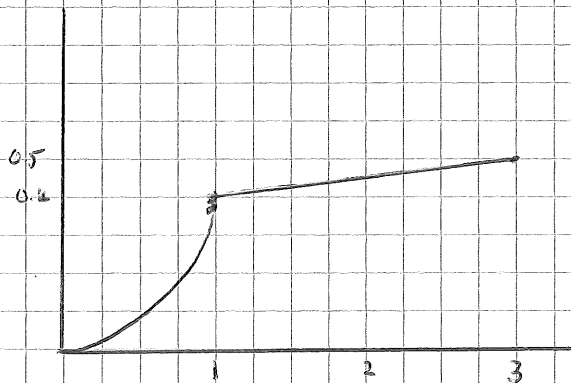
Do not Reject H_0

Not enough evidence at 1% level to suggest the teacher is underestimating H/W times



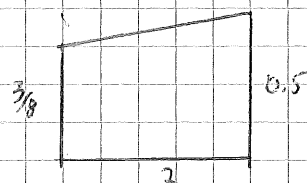
b) H/W times are Normally Distributed

6) a)



b) $P(T \geq 1)$

= Area of Trapezium



$$\begin{aligned} &= \frac{1}{2} (3/8 + 0.5) \times 2 \\ &= 7/8 \end{aligned}$$

c) i) From b), $F(1) = 1 - 7/8 = 1/8$

This needs adding on!

Need: $\int 1/16 t + 5/16 dt$

$= \left[1/32 t^2 + 5/16 t \right]_1^t$

$= 1/32 t^2 + 5/16 t - 1/32 - 5/16$

So: $F(t) = 1/32 t^2 + 5/16 t - 1/32 - 5/16 + 1/8$

$= 1/32 t^2 + 5/16 t - 7/32$

$= 1/32 (t^2 + 10t - 7)$

ii) For median $F(t) = 0.5$

$\therefore 1/32 (t^2 + 10t - 7) = 0.5$

$t^2 + 10t - 7 = 16$

$t^2 + 10t - 23 = 0$

Does not factorise: $t = \frac{-10 \pm \sqrt{10^2 - 4 \times 1 \times (-23)}}{2}$

$t = \frac{-10 \pm \sqrt{192}}{2} = -5 \pm \sqrt{48}$

As t between 0 and 3

median = $-5 + \sqrt{48} = 1.9282...$

① a) H_0 : no association between maths performance at KS3 & GCSE (Independent)

H_1 : there is an association (Non-Independent)

	Expected				χ^2	$\frac{ O_i - E_i ^2}{E_i}$			
	A	B	C	Below C		A	B	C	Below C
8	63.55	51.25	46.33	43.87	8	0.1983	0.2764	0.0097	0.0173
7	44.64	36	32.54	30.82	7	2.4043	0.6644	0.0733	0.7527
6	46.81	31.75	34.13	32.31	6	0.9407	0.0017	0.0222	1.0005

$\chi^2 = 6.1897$ (Test Statistic)

CRITICAL VALUE: $v = (4-1)(3-1) = 6$, $\chi^2_{(0.05)}(6) = 10.645$



$$6.1897 < 10.645$$

Do NOT reject H_0

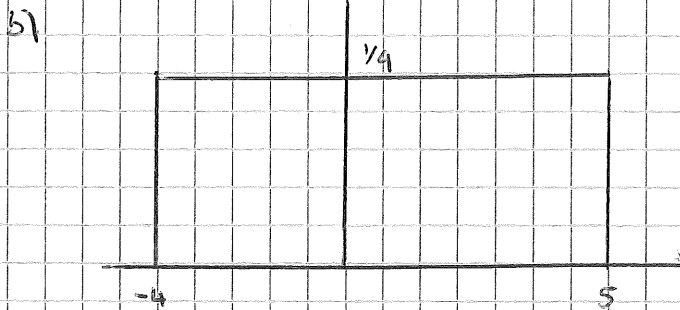
Not enough evidence at 90% level to suggest an association between KS3 & GCSE performance.

b) More students who got Level 7 achieved and A than expected.

8) a) $x \leq -4 \rightarrow 0$ and $x \geq 5 \rightarrow 0$

Let $y = \frac{x+4}{4}$, $dy/dx = 1/4$

$$\therefore f(x) = \begin{cases} 1/4 & -4 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$$



c) $P(x > 2) = (5-2) \times 1/4 = 3/4$

d) Mean = $\frac{5 + (-4)}{2} = 1/2$

Variance = $1/12 (b-a)^2 = 1/12 (-4-5)^2 = 6.75$